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Natural Transition in Transonic Flows Using an Efficient Temporal/Spatial Formulation

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Transition prediction capabilities are critical to wing design for minimizing skin friction while retaining the desired lift characteristics. A new stability analyzer is presented that combines beneficial aspects of both temporal and spatial formulations, to result in an efficient stability analyzer. This technique was validated by comparison with the recognized Office National d'Etudes et de Recherches Aérospatiales/Centre d'Etudes et de Recherches de Toulouse method and observed transition locations. It is demonstrated, through numerical examples, that the temporal frequency tracking procedure is equivalent to its spatial counterpart.

Nomenclature

A = amplitude
c = chord length, m
f = frequency, Hz
M = Mach number
$[M_i]$, $[P_s]$, $[P_t]$ = eigenvalue problem matrices
N = number of grid points
n = amplification factor
P = mean pressure
p = instantaneous pressure
Q = mean flow quantity
q = instantaneous flow quantity
Re = Reynolds number, $U_e c/\nu^*$
t = time
U, V = mean velocity components
u, v, w = instantaneous velocity components
V_g = group velocity
x, y, z = local Cartesian coordinates
α, β = wave numbers
γ_i = spatial growth rate along the path of propagation
δ_e = boundary-layer thickness, m
δ^* = boundary-layer displacement thickness, m
η = transformed normal coordinate
ν^* = kinematic viscosity
τ = instantaneous temperature
Φ = generalized eigenvector
ψ = wave orientation
ω = complex frequency

Subscripts
e = dimensional quantities at the edge of the boundary layer
r, i = real and imaginary parts

Superscript
$\sim$ = perturbation quantities

I. Introduction

THE potential for reduced frictional losses represented by sustained laminar flow over the airfoil has long been recognized. In this context, natural laminar flow (NLF) airfoil technology finds a promising field of application. However, drag prediction for NLF wings presents one of the most demanding problems posed in applied aerodynamics; mainly because it hinges on obtaining an accurate estimate of the location of laminar/turbulent transition of the boundary layer. In the industrial environment, the development of NLF airfoils requires a cost-effective design tool capable of delivering a good approximation to the location of transition. The linear stability theory, along with the empirical e^n method, provides an appropriate framework for transition predictions over wings in the cruise regime. Among others, Mack,[1] Malik and Orszag,[2] and Cebeci and Stewartson,[3] have developed numerical methods for the solution of the linear stability equations. Mack[1] used an initial value method (IVM) for the solution of the spatial stability equations. Malik and Orszag[2] considered IVMs computationally slow and proposed a more efficient solver for the solution of the temporal stability equations based on a boundary value method (BVM). Cebeci and Stewartson[3] and Arnal and Juillen[4] proposed BVMs for the solution of the spatial stability equations. The group of the J.-A. Bombardier Chair has recently developed a linear stability analyzer [Stabilité de la Couche Limite Compressible (SCOLIC)] based on the temporal formulation.[5] Pressure distributions were taken from available experimental data. The boundary layer was predicted using the MAIN computer code.[6]

This article presents an extension of SCOLIC: a hybrid method was developed[7] to exploit the strengths and advantages of both the temporal and spatial formulations, resulting in a very efficient stability analyzer. The resulting stability analyzer has been validated through comparisons with available numerical solutions and experimental data.

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II. Linear Stability Theory

A. General Formulation

The customary notation is adopted for a general three-dimensional, compressible boundary layer. For a local Cartesian coordinate system, the flow variables are assumed to be of the form

$$q(x, y, z, t) = Q(y) + \tilde{q}(y)\exp[i(\alpha x + \beta z - \omega t)] \quad (1)$$

where $Q(y)$ is the mean laminar profile and the tilde denotes the perturbation quantities for any of the velocity components, pressure, or temperature. α and β and ω are generally complex.

The problem is formulated using the parallel flow assumption $[V(y) = 0]$, which implies that the pressure in the boundary layer is constant in the normal direction ($\partial P/\partial y = 0$). The definitions of Eq. (1) are substituted into the momentum, state, and energy equations. The nonhomogeneous terms are eliminated by subtracting the laminar mean flow solution from the equations. Neglecting the nonlinear and higher-order terms yields a homogeneous system of five linear, second-order ordinary differential equations. This mathematical model is, in general, not amenable to analytical solution techniques. Consequently, the solution procedure proposed here is based on numerical techniques.

There are several discretization strategies suited for the solution of the linear stability equations. In our approach, a finite difference method with a staggered mesh for pressure was chosen. It is convenient to map the physical domain $0 \leq y \leq \delta_e$ into a transformed domain $0 \leq \eta \leq N - 1$, similar to that proposed by Malik and Orszag.[2] This transformation was proposed to allow a fine resolution in the critical layer of the physical domain while using a uniform grid spacing in the transformed domain. The main benefit is that the finite difference expressions for the first and second derivatives in the transformed domain are then second-order accurate. With this mapping, the discretized system can be expressed as

$$(\omega[M_1] + \alpha[M_2] + \alpha^2[M_3] + \beta[M_4] + \beta^2[M_5] + [M_6])\{\Phi\} = 0 \quad (2)$$

where Φ is the perturbation vector:

$$\Phi = (\alpha\tilde{u} + \beta\tilde{w}, \tilde{v}, \tilde{p}, \tilde{\tau}, \alpha\tilde{w} - \beta\tilde{u}) \quad (3)$$

$\{\Phi\}$ is a $(5N - 9)$ coefficient vector representing the discrete eigenvectors, $[M_i]$ are $(5N - 9) \times (5N - 9)$ coefficient matrices.

The disturbances vanish at the wall and in the freestream, except for the pressure fluctuations. The staggering obviates the need for direct treatment of the pressure at the boundaries. Hence, in the framework of the linear stability theory, the boundary-layer stability analysis reduces to a homogeneous eigenvalue problem for which nontrivial solutions exist only for certain combinations of α, β, and ω. In general, α, β, and ω are complex numbers, corresponding to six real parameters. Solution of the eigenvalue problem provides a relation for only two of these so that the resulting system of equations is not mathematically closed. It is thus customary and necessary to make some basic assumptions about the nature of α, β, and ω.

B. Temporal Stability Theory

In the temporal stability theory, α and β are assumed real. For given values of α and β, the system of equations is amenable to a linear eigenvalue problem in ω:

$$[P_t]\{\Phi\} = \omega[M_1]\{\Phi\} \quad (4)$$

where

$$[P_t] = -(\alpha[M_2] + \alpha^2[M_3] + \beta[M_4] + \beta^2[M_5] + [M_6]) \quad (5)$$

The real part of the temporal eigenvalue ω_r is the frequency, and the imaginary part ω_i is the temporal growth rate.

The temporal stability theory yields an eigenvalue problem that can be solved using classical numerical algorithms. These methods can be divided into two categories: 1) global and 2) local methods. Global methods are used to obtain the complete eigenvalue spectrum. These are quite expensive in terms of computational resources, but they do not need any initial guess for the eigenvalue. Local methods are more efficient and generally more accurate, but require an initial guess for the eigenvalue.

The SCOLIC implementation of the temporal stability theory calls for a coarse grid solution to a global calculation, using the QZ algorithm,[8] performed to yield a spectrum of $(5N - 9)$ eigenvalues. A suitable candidate is then selected as an initial guess for a local method applied to a finer grid for the purpose of improving the solution estimate. Local calculations use an inverse Rayleigh iteration procedure.[2]

C. Spatial Stability Theory

In the spatial stability theory, ω is assumed real and given, α and β are complex. For given values of β, the resulting system of equations can thus be expressed as a nonlinear eigenvalue problem in α:

$$[P_s]\{\Phi\} = (\alpha[M_2] + \alpha^2[M_3])\{\Phi\} \quad (6)$$

where

$$[P_s] = -(\omega[M_1] + \beta[M_4] + \beta^2[M_5] + [M_6]) \quad (7)$$

The real part of the spatial eigenvalue α_r yields the wave number, and the imaginary part α_i yields the spatial growth rate.

The resulting eigenvalue problem is nonlinear in α and it can be solved using, for example, the companion matrix method.[9] This standard approach uses a simple transformation, first proposed by Bridges and Morris,[9] to linearize all terms. The resulting form of the eigenvalue problem is suited to the standard QZ algorithm for computing the entire eigenvalue spectrum. The order of this eigenvalue problem is almost twice as large as the order of that corresponding to the temporal formulation, which makes the global calculations in the context of the spatial formulation much longer to perform.

Such a global method was considered inefficient for engineering calculations. Therefore, the inverse Rayleigh iteration method was preferred and applied to the following linearized problem:

$$[P_s]\{\Phi\} = \alpha([M_2] + \alpha_{\text{guess}}[M_3])\{\Phi\} \quad (8)$$

where α_{guess} represents the needed initial guess for the eigenvalue of interest. The solution is obtained iteratively.

In a traditional spatial method, the initial guess is provided by a global calculation using an appropriate solution procedure, such as the companion matrix method. In this article, a more efficient procedure, presented in Sec.III.C, is proposed for the calculation of the initial guess.

III. Transition Prediction

The main assumption in linearized transition prediction is that there exists a critical amplification of the disturbances at the transition location. The amplitude ratio A/A_0 can be calculated assuming that the growth of the disturbance from its initial amplitude A_0 to the critical value A_{crit} can be predicted by the linear stability characteristics. The ratio A/A_0, or the more commonly used amplification factor, $n = \ell n(A/A_0)$, is therefore calculated by integrating γ_i, which is parallel to the real part of $\boldsymbol{V}_{gr}$:

$$n(s) = \int_{s_0}^{s} -\gamma_i \, \mathrm{d}s \quad (9)$$

where s is a point of the path of the disturbance, and s_0 is the point of inception of the perturbation ($\gamma_i = 0$). The real part of the group velocity is given by

$$V_{gr} = \mathrm{Re}\left(\frac{\partial \omega}{\partial \alpha}, \frac{\partial \omega}{\partial \beta}\right) \tag{10}$$

For low background turbulence levels in two-dimensional flows, it has been observed experimentally that n is approximately 9 at transition. This correlation of the amplification factor with transition location is the basis of the e^n method. The n-factor calculation is usually done under the constraint of constant dimensional frequency:

$$f = \frac{V_e}{\delta^*}\frac{\omega_r}{2\pi} \tag{11}$$

A finite number of frequencies are selected, and an n factor is calculated for each of them. The frequency that reaches the critical n factor first is considered the most relevant frequency for the prediction of transition location.

A. Temporal Frequency Tracking

In the temporal stability theory, there are four independent parameters: α, β, ω_r, and ω_i. The temporal eigenvalue problem provides two real relations. In the calculation of the n factor, the frequency ω_r is given (based on an appropriate frequency selection criterion). The additional relation (or constraint) needed to close the problem is provided by the maximization of the temporal growth rate ω_i. These four real relations are the basis for the frequency-tracking procedure implemented in SCOLIC.

The spatial amplification factor in the direction of the real part of the group velocity is obtained using Gaster's relation[10]:

$$\gamma_i = -(\omega_i/|V_{gr}|) \tag{12}$$

The evaluation of γ_i involves the knowledge of the group velocity. In the temporal formulation, where only ω is a complex number, the real part of the group velocity is given by

$$V_{gr} = \left(\frac{\partial \omega_r}{\partial \alpha}, \frac{\partial \omega_r}{\partial \beta}\right) \tag{13}$$

The basis of the temporal frequency tracking is the maximization of ω_i for a given ω_r. First, the desired frequency, ω_r^{SPEC} is found, and then ω_i is maximized along this constant frequency contour. To find the desired frequency, $\Delta\alpha$ and $\Delta\beta$ are given by

$$\Delta\alpha = \frac{\Delta\omega_r}{|V_{gr}|^2}\frac{\partial \omega_r}{\partial \alpha} \tag{14}$$

$$\Delta\beta = \frac{\Delta\omega_r}{|V_{gr}|^2}\frac{\partial \omega_r}{\partial \beta} \tag{15}$$

where

$$\Delta\omega_r = \omega_r^{\mathrm{SPEC}} - \omega_r \tag{16}$$

Once the specified frequency is reached, ω_i is maximized along the constant frequency contour. The appropriate expressions for $\Delta\alpha$ and $\Delta\beta$ are obtained from the complex dispersion relation:

$$\Delta\beta = \frac{1}{|V_{gi}|}\left[\frac{\partial \omega_i}{\partial \beta} - \left(\frac{\partial \omega_r}{\partial \beta}\bigg/\frac{\partial \omega_r}{\partial \alpha}\right)\frac{\partial \omega_i}{\partial \alpha}\right] \tag{17}$$

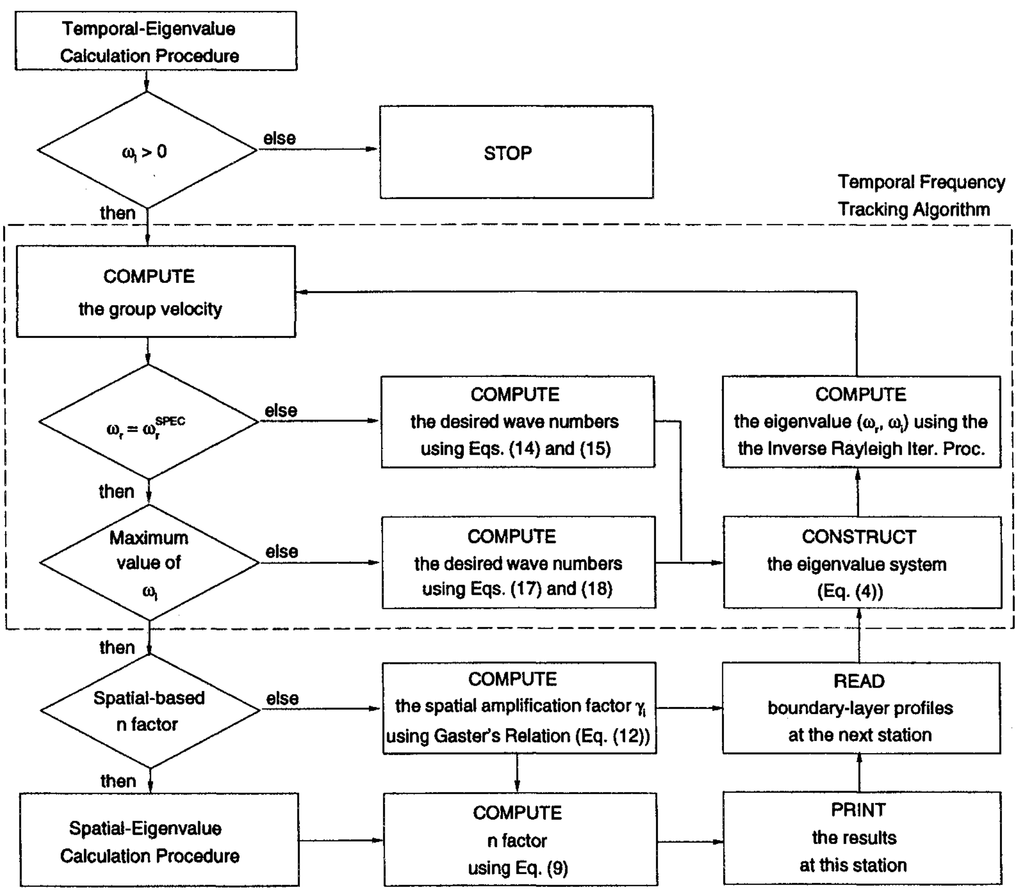

Fig. 1 Overall algorithm for the hybrid method.

$$\Delta\alpha = \left(\frac{\partial \omega_r}{\partial \beta} \bigg/ \frac{\partial \omega_r}{\partial \alpha}\right) \Delta\beta \qquad (18)$$

The algorithm for the frequency tracking is the flow chart contained within the dashed-line box presented in Fig. 1.

B. Spatial Frequency Tracking

In the spatial stability theory, there are five independent parameters: α_r, α_i, β_r, β_i, and ω. The spatial eigenvalue problem, the given frequency ω, and the optimization of the spatial growth rate provide four real relations. The fifth real relation is obtained from the constraint that

$$\frac{\partial \alpha}{\partial \beta} \in \text{Real} \qquad (19)$$

This relation was derived by Cebeci and Stewartson[3] and Nayfeh.[11] These five real relations can be used to derive a spatial frequency tracking method. The spatial amplification factor in the x direction, α_i, is a direct result of the nonlinear eigenvalue problem of the spatial stability theory. The spatial amplification factor in the direction of the real part of the group velocity, needed for the calculation of the n factor, is given by

$$\gamma_i = \frac{1}{|V_{gr}|}\left[\alpha_i \, \text{Re}\left(\frac{\partial \omega}{\partial \alpha}\right) + \beta_i \, \text{Re}\left(\frac{\partial \omega}{\partial \beta}\right)\right] \qquad (20)$$

The real part of the group velocity is given by Eq. (10).

C. Proposed Hybrid Method

The advantage of the hybrid method is that the temporal formulation is used to obtain a first guess for the spatial growth rate. The linearity of the temporal system results in time saving with respect to the nonlinear spatial system. In the proposed hybrid method, global calculations are performed using the temporal formulation: for given real α and β, the temporal eigenvalues ω_r and ω_i can be obtained. The temporal eigenvalues obtained with the global calculation are then transformed into spatial eigenvalues using appropriate relations that exist between the temporal and the spatial formulations.[10,11] This provides the initial guess for the local solution of the linearized spatial eigenvalue problem, Eq. (8).

In the case of an infinite swept wing, the approximation $\beta_i = 0$ can be used, and assuming that the imaginary part of the group velocity is much smaller than the real part, the relation between the temporal and spatial formulations is given by

$$\alpha_i = -\left(\omega_i \bigg/ \frac{\partial \omega_r}{\partial \alpha}\right) \qquad (21)$$

Eq. (21) is the relation used in the hybrid method to calculate the initial guesses for the spatial eigenvalues from the results of the global calculation performed in the context of the temporal formulation.

In the hybrid method, the instability tracking is done in the context of the temporal formulation. Once the appropriate disturbance is found, a local calculation based on the spatial formulation is achieved to obtain directly the spatial growth rate needed in the calculation of the n factor. The algorithm used by the hybrid method for the calculation of the n factor is illustrated in Fig. 1. This algorithm assumes that the group velocity based on the temporal formulation is a good approximation of the spatial group velocity.

The main differences between the temporal and spatial frequency tracking procedures are the following:

1) Maximization is applied on the temporal growth rate in the former, while in the latter the spatial growth rate is optimized.

2) An additional constraint, Eq. (19), is applied in the spatial frequency tracking.

It can be shown that the frequency trackings used in the hybrid and spatial methods are equivalent. According to Nayfeh,[11] for a parallel flow, the additional constraint, Eq. (19), reduces to

$$\frac{\partial \omega}{\partial \alpha} \bigg/ \frac{\partial \omega}{\partial \beta} \in \text{Real} \qquad (22)$$

This equation stipulates that for a physical problem, the ratio

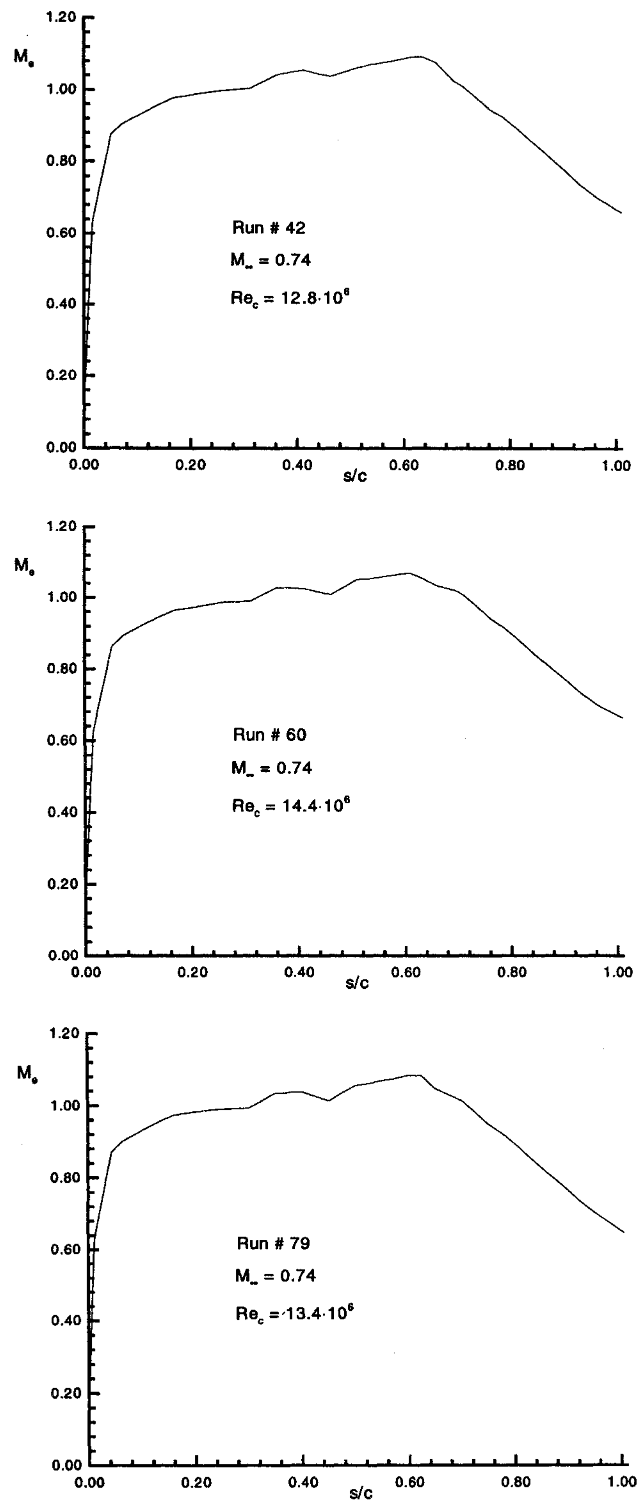

Fig. 2 Experimental Mach number distributions.

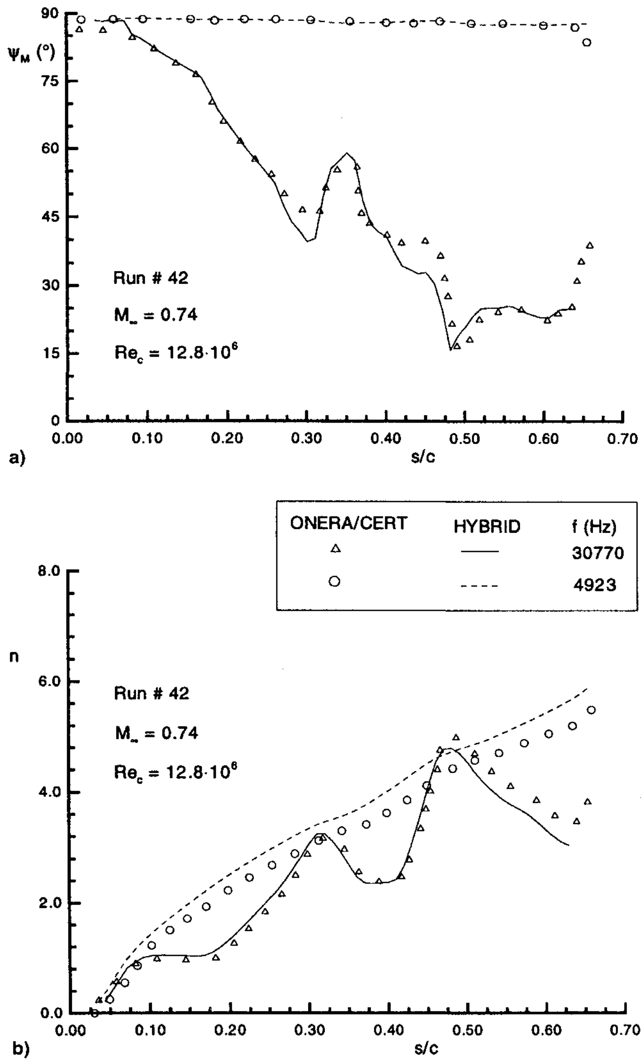

Fig. 3 ***n*-factor calculations: a) orientation of the most amplified disturbance and b) *n*-factor distribution.**

of the group velocity components must be real. In the context of the temporal formulation, Eq. (22) can be recast as

$$\frac{\partial \omega_r}{\partial \alpha} \Big/ \frac{\partial \omega_r}{\partial \beta} = \frac{\partial \omega_i}{\partial \alpha} \Big/ \frac{\partial \omega_i}{\partial \beta} \tag{23}$$

which corresponds to the maximization of ω_i for a given ω_r. Therefore, the maximization of ω_i for a constant ω_r of the temporal frequency tracking procedure is equivalent to the saddle-point condition used in the spatial frequency tracking. However, the maximization of ω_i for a constant ω_r does not ensure that the spatial growth rate in the direction of the real part of the group velocity is maximized. This question is of physical relevance in cases where multiple local maxima of ω_i exist for a given frequency ω_r (e.g., regions of the flow where both crossflow and Tollmien–Schlichting instabilities occur). In the context of the temporal formulation as is used in the hybrid method, the local maximum of ω_i corresponding to the maximum value of the spatial growth rate is chosen by numerical inspection so that the results obtained with the proposed hybrid method should be the same (within numerical accuracy) as those obtained with a traditional spatial formulation. Furthermore, the proposed hybrid method is believed to be more efficient than the traditional spatial formulation.

The main advantages of the proposed hybrid method are the following:

1) The initial guesses for the spatial eigenvalues are obtained using a global calculation with the temporal formulation that requires less CPU time and memory than the corresponding spatial problem.

2) Gaster's relation is not needed for the *n*-factor calculations: the spatial growth rate is a direct result of the linearized spatial eigenvalue problem.

IV. Results

To demonstrate the capability of the proposed hybrid method, stability/transition calculations over the suction side of a 15-deg swept tapered wing with an AS409 cross section were undertaken. Experimental pressure distributions and transition locations were obtained in the T2 transonic tunnel of ONERA/CERT.[12–14] Cebeci et al.[13] and Niethammer[14] have also presented stability/transition calculations over this wing using the spatial theory. To avoid fully three-dimensional stability/transition analysis, their calculations were performed under the approximation of an infinite swept wing having the mean sweep angle of the actual swept tapered wing. The results presented in this article correspond to experimental runs no. 42, 60, and 79.[12] The experimental Mach number distributions are given in Fig. 2. Some physical irregularities are present on the test model as can be seen from the Mach number distributions near $x/c = 0.3$ and 0.5.

The results for run no. 42 are presented in Fig. 3, where the predictions of the hybrid method are compared with the solutions obtained using the ONERA/CERT method.[13,14] Run no. 42 was for a stagnation temperature and pressure of 145 K

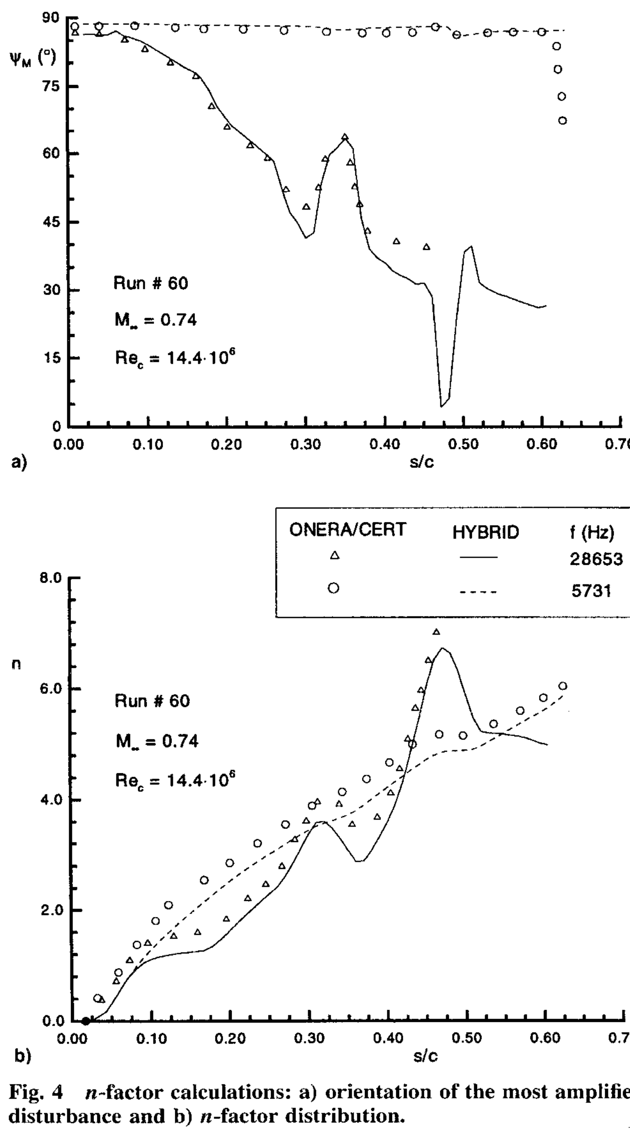

Fig. 4 ***n*-factor calculations: a) orientation of the most amplified disturbance and b) *n*-factor distribution.**

and 2 bars, respectively, and a chord Reynolds number of 12.8 $\times$ 10^6. Figure 3a shows the orientation of the most amplified disturbance as a function of the distance from the attachment line s/c for two selected frequencies. For the purpose of comparison, the frequencies used in this study are the ones selected in Refs. 13 and 14. The results of the proposed hybrid method and the ONERA/CERT method are in very good agreement. The orientation of the most amplified low-frequency disturbance (f = 4923 Hz) is almost constant along the wing with a value near 88 deg, indicating a pure crossflow instability. In this case, the temporal frequency tracking procedure used in the hybrid method produces results identical to the spatial procedure of the ONERA/CERT method. In the case of the higher frequency (f = 30770 Hz), large variations of the most amplified disturbance orientation along the wing are predicted by both methods. Near the attachment line, the disturbance is mainly in the direction of the crossflow instabilities. The large variations in the most amplified disturbance orientation are the result of the locally adverse pressure gradients induced by the presence of small hollows in the model.[14] The hybrid method predicts these large variations in satisfactory agreement with the solution obtained using the ONERA/CERT method. These successful comparisons of the orientation of the most amplified disturbance strongly suggest that the frequency tracking procedure of the temporal formulation is equivalent to its spatial counterpart. n-factor calculations are presented in Fig. 3b. Here, again, the present results are very close to the ones obtained with the ONERA/CERT method.

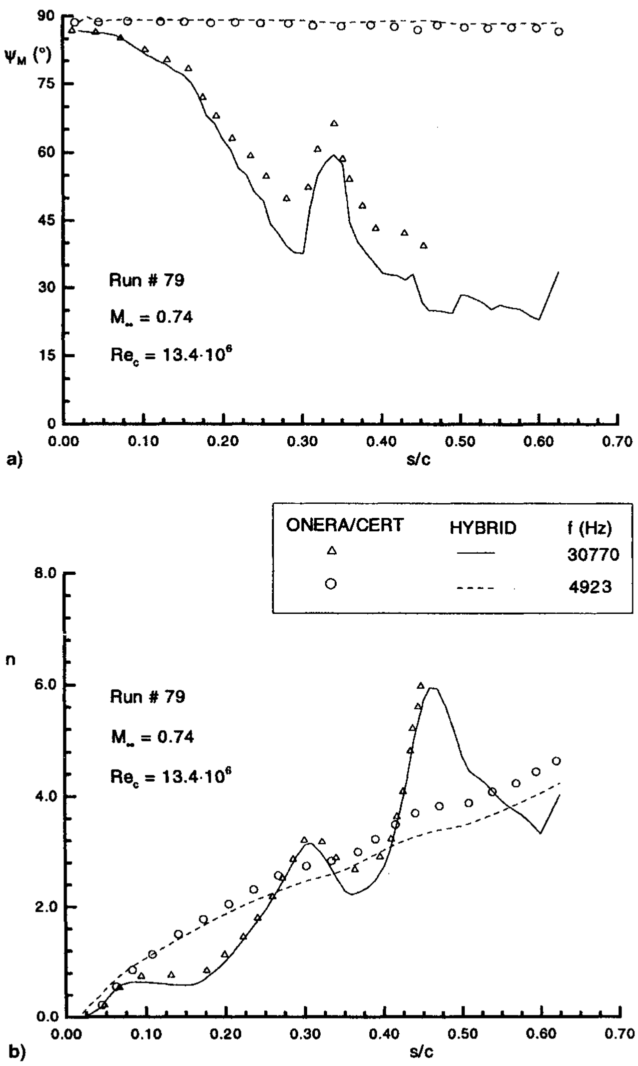

Fig. 5 n-factor calculations: a) orientation of the most amplified disturbance and b) n-factor distribution.

The results concerning run no. 60 are illustrated in Fig. 4. The stagnation temperature, pressure, and the Reynolds number for this case are 134 K, 2 bars, and 14.4 $\times$ 10^6, respectively. The orientation of the most amplified disturbance predicted by the proposed hybrid method and the ONERA/CERT method (see Fig. 4a) are again in very good agreement, supporting the validity of the frequency tracking procedure implemented in the proposed hybrid method. Predictions with the hybrid method for the n factors are lower than those of the ONERA/CERT method. For any given frequency, the difference is nearly constant along the wing (about 0.2 for f = 28,653 Hz and 0.4 for f = 5731 Hz). This nearly constant difference can be attributed to a slight delay of the point of inception predicted by the hybrid method with respect to the one calculated by the ONERA/CERT method. This difference, however, is expected to be related to the size of the marching step near the leading edge during the laminar boundary-layer calculations. Using a similar mesh should result in a closer agreement between the two methods.

The influence of larger stagnation temperature and pressure on the stability behavior is illustrated in run no. 79 for which the stagnation temperature, pressure, and the Reynolds number are 164 K, 2.5 bars, and 13.4 $\times$ 10^6, respectively. The comparisons, presented in Fig. 5, show again good agreement, adding confidence to the validity of the proposed hybrid method.

The prediction of the critical n factor for runs no. 42 and 79 has been conducted using the frequency selection strategy proposed in Ref. 15. The results are presented in Fig. 6. The experimental transition location for run no. 42 is around x/c =

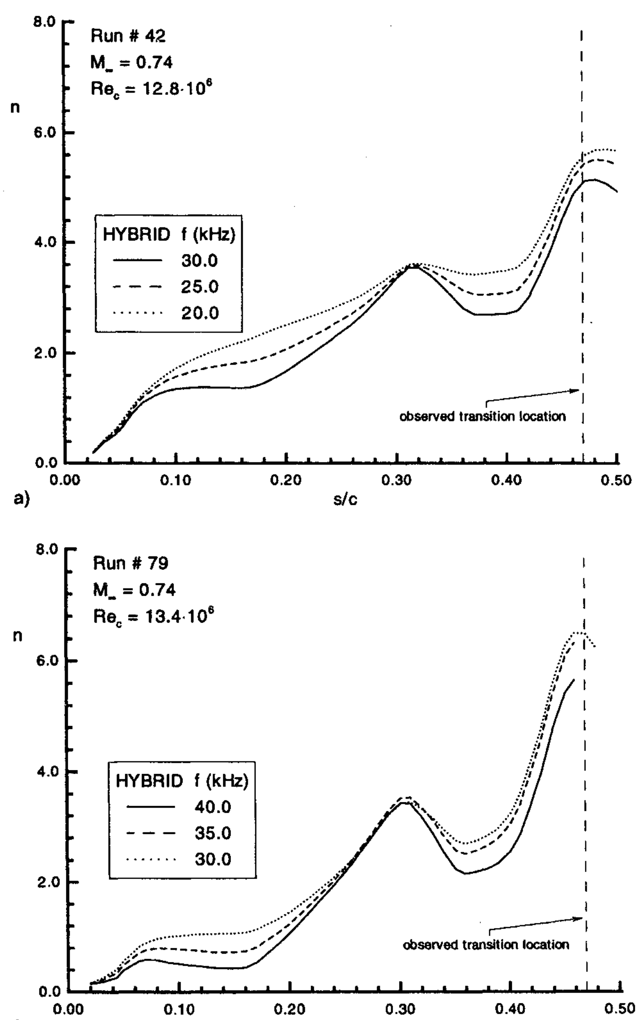

Fig. 6 Critical n-factor calculations.

0.47. Based on the results of Fig. 6a, the critical n factor is around 5.6. Experimental transition location is not available for run no. 79, but since its drag coefficient is similar to the one of run no. 42, it may be assumed that transition occurs near $x/c = 0.47$.[13,14] Based on the results of Fig. 6b, and using this assumed transition location, the critical n factor for run no. 79 is 6.5. The proposed hybrid method is certainly adequate for transition predictions, considering that the critical n factor for the T2 transonic tunnel is between 7–8.[13,14] The critical n factor for run no. 60 is not predicted since the transition for this run was triggered by ice crystals, a phenomenon that is not modeled in this analysis.

V. Conclusions

The major contribution of this work is the implementation of the spatial formulation and its original inclusion in SCOLIC, yielding a new hybrid method. The main idea in the proposed hybrid method is to join the advantages and strengths of the two formulations resulting in an effective method.

1) The global calculations are performed on the linear eigenvalue problem of the temporal formulation since the order of the matrices involved is almost half the order of those involved during the global solution of the nonlinear eigenvalue problem associated to the spatial formulation.

2) The selection of the most critical frequency is done using the temporal formulation, since the frequency is part of the solution in this formulation, making the prediction of the frequency of the most amplified disturbance an easier task than in the context of the spatial formulation.

3) The tracking of the instability during the n-factor calculations is conducted using the temporal formulation, since it involves four parameters instead of the five parameters associated to the spatial formulation.

4) The spatial growth rate used in the calculation of the n factor is obtained by a local solution of the linearized spatial eigenvalue problem, eliminating reliance on Gaster's relation.

The proposed hybrid method has been validated through comparisons with available numerical solutions and observed transition locations.

Acknowledgments

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